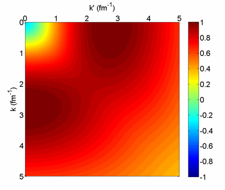
Operator Evolution via the Similarity Renormalization Group



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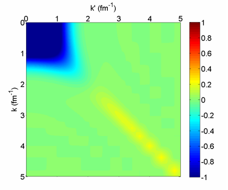
June 23, 2010



THE OHIO STATE UNIVERSITY

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SRG: A Brief Review

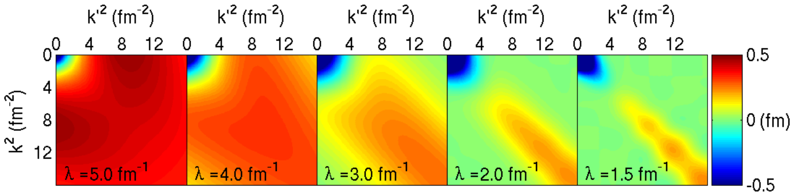
- **The Similarity Renormalization Group (SRG)**

  → provides a means to systematically evolve computationally difficult **potentials and operators**

$$\frac{dO_s}{ds} = [\eta_s, O_s] = [[T_{rel}, H_s], O_s], \quad \Longleftrightarrow \quad O_s = U_s O_{s=0} U_s^\dagger$$

Where $U_s = \sum_\alpha |\psi_\alpha(s)\rangle \langle \psi_\alpha(0)|$

- Hamiltonian Operator → driven toward diagonal or decoupled form



$^3S_1$ **AV18 Evolution**

- Can unitarily evolve operators consistent with any initial potential

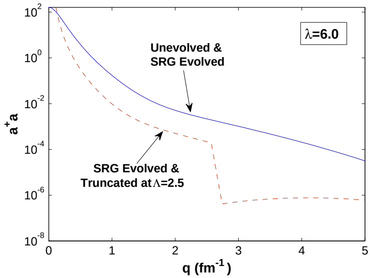
- Issues: – Decoupling – Many-body Operators
  – Basis issues – Factorization

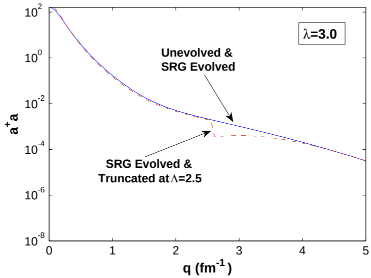
Note:

$$\lambda = \frac{1}{s^{1/4}}\ \text{fm}^{-1}$$

Demonstration of Decoupling In Expectation Values

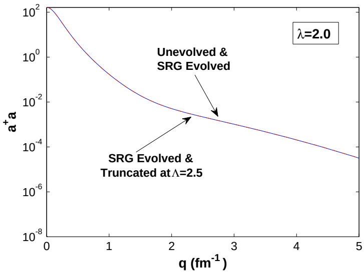
- **Evolve** Hamiltonian & operators **to λ in full space** $\rightarrow$ **TRUNCATE at** Λ:





- Momentum distribution
  - Calculated with AV18 potential.
  - $\Lambda = 2.5\,\text{fm}^{-1}$
    $\lambda = 6.0\,\text{fm}^{-1}, 3.0\,\text{fm}^{-1}$, and $2.0\,\text{fm}^{-1}$
- Decoupling **for all q** is successful when $\lambda < \Lambda$

  $\Rightarrow$ Expectation values reproduced in truncated basis



Many-Body evolution of Operators

- **Many-body evolution** with operators normal ordered in the vacuum:

$$\frac{d\widehat{O}_s}{ds} = [[T_{\rm rel}, H_s], \widehat{O}_s] \Longrightarrow \left[\left[\sum_{ij} T_{ij} a_i^\dagger a_j, \sum_{i'j'} T_{i'j'} a_{i'}^\dagger a_{j'} + \frac{1}{2}\sum_{pqkl} V_{pqkl|s} a_p^\dagger a_q^\dagger a_l a_k + \cdots\right], \widehat{O}_s\right]$$

$\rightarrow$ Only one non-vanishing contraction in the vacuum: $\overline{a_i a_j^\dagger} = \delta_{ij}$

- A general operator $\widehat{O}$ for an A-body system can be written as

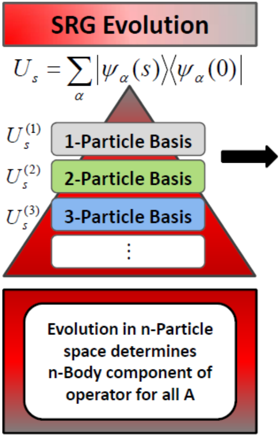
$$\widehat{O} = \widehat{O}^{(1)} + \widehat{O}^{(2)} + \widehat{O}^{(3)} + \cdots + \widehat{O}^{(A)}$$

where the $\widehat{O}^{(i)}$ label the $i = 1, 2, 3, ..., A$-body components
– SRG operator $\widehat{O}_s$ will have contributions for all n so that $\widehat{O}^{(n)} \neq \widehat{O}_s^{(n)}$.

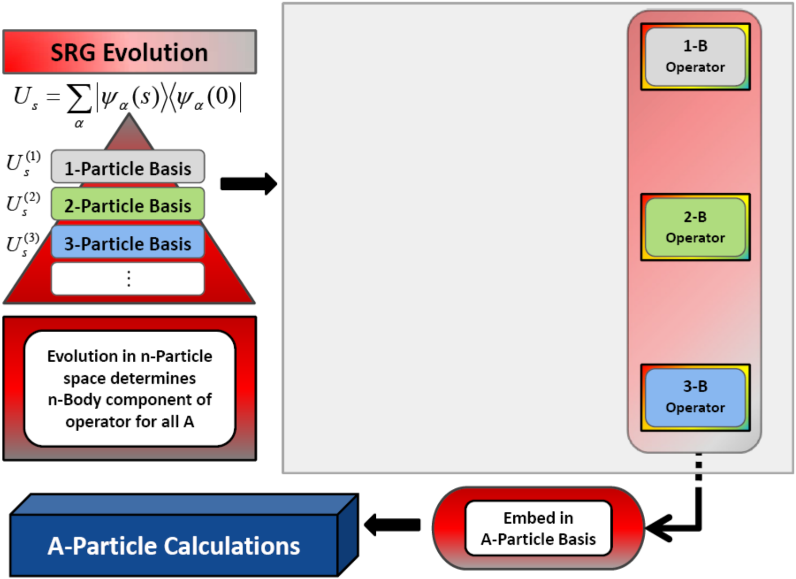
- Expanding commutators and making contractions, one finds:

$\rightarrow$ Evolution of an operator is fixed in each n-particle subspace
$\rightarrow$ **How do we deal with this in practice?**

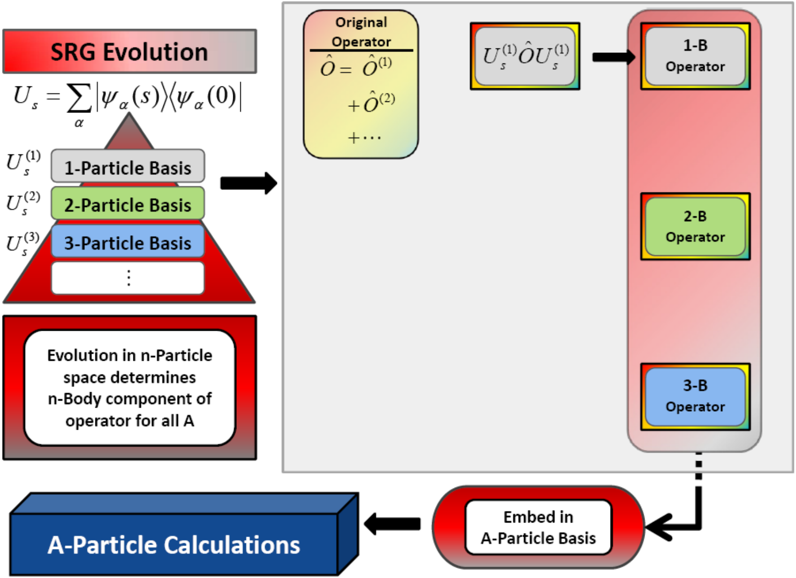
Operator Evolution & Extraction Process



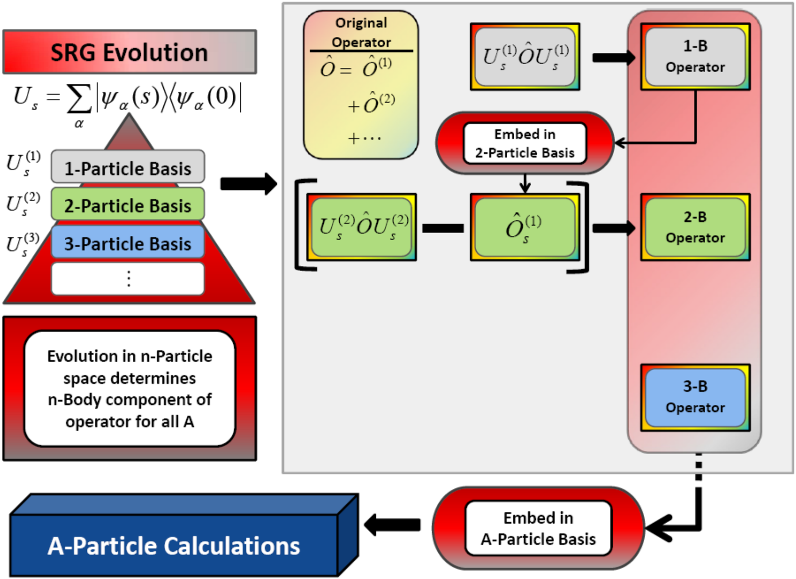
Operator Evolution & Extraction Process



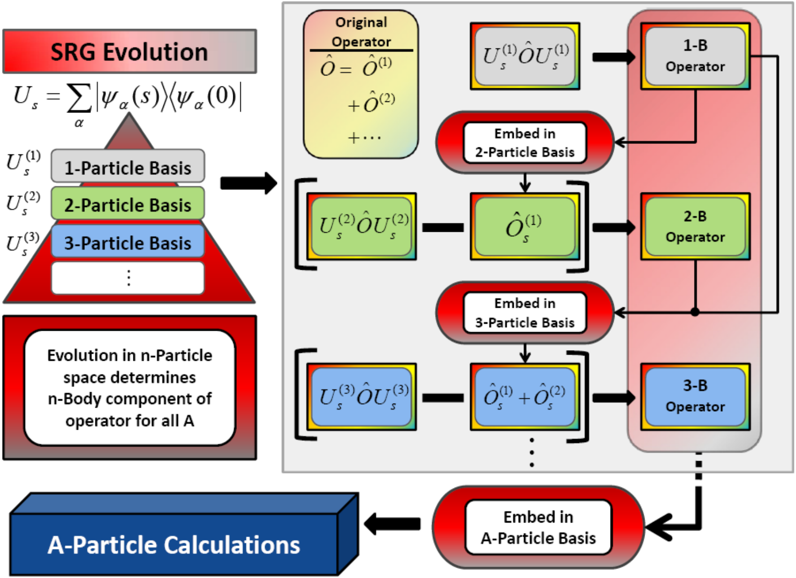
Operator Evolution & Extraction Process



Operator Evolution & Extraction Process

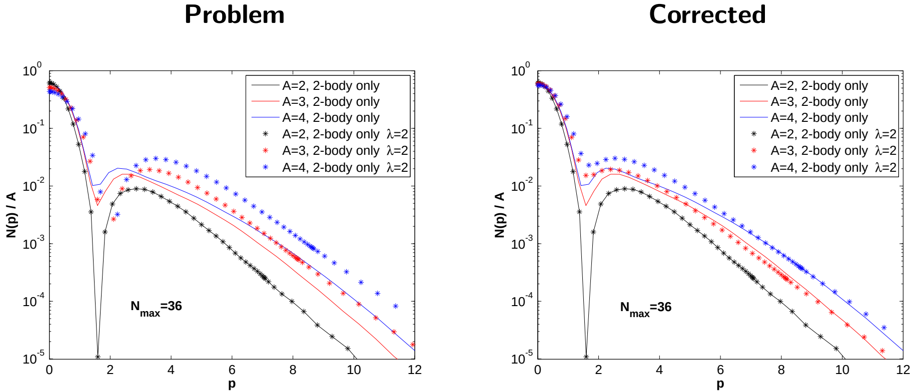


Operator Evolution & Extraction Process



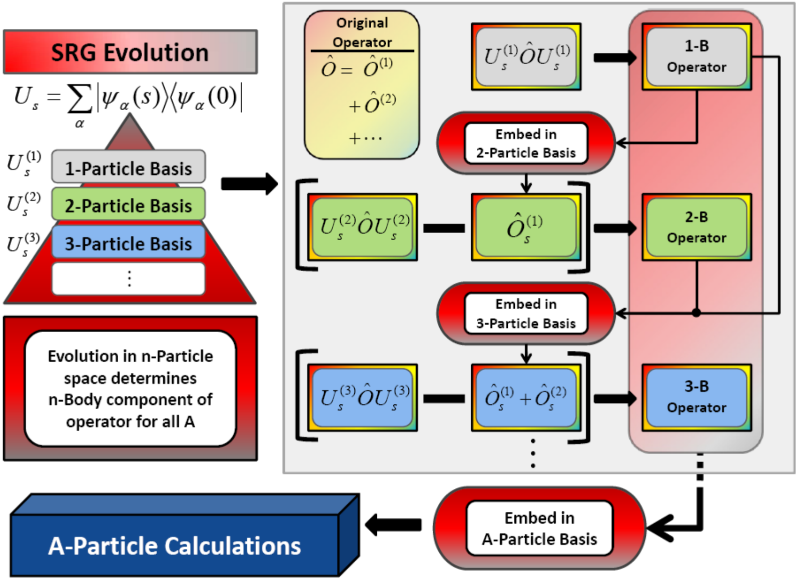
Embedding Other Operators

- Consider the following momentum distributions in 1D model . . .

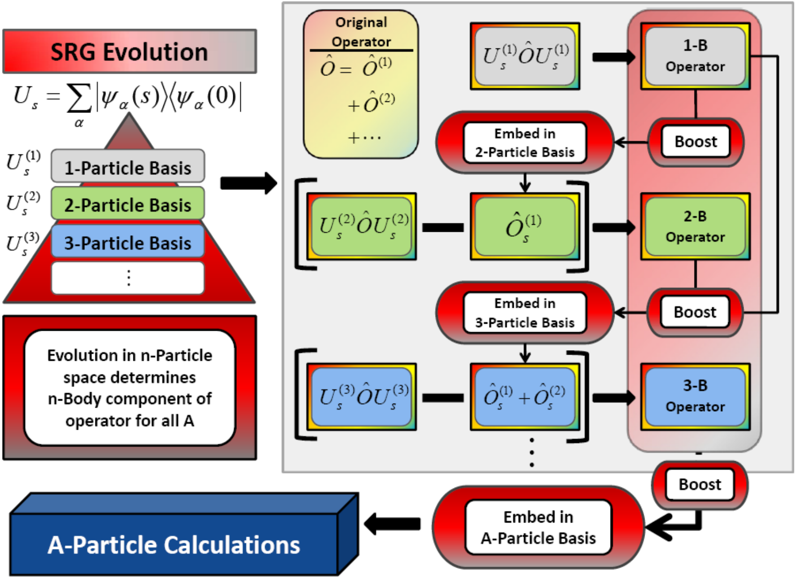


Solution: Operators must be "boosted" to embed into A-particle space

- Revised Embedding Process . . .

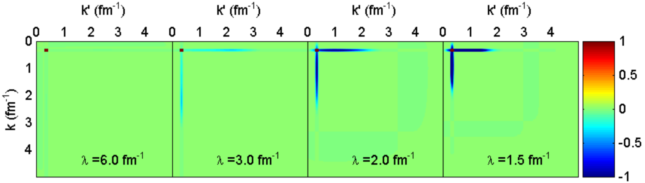

SRG Evolution
$U_s = \sum_\alpha |\psi_\alpha(s)\rangle\langle\psi_\alpha(0)|$
$U_s^{(1)}$
1-Particle Basis
$U_s^{(2)}$
2-Particle Basis
$U_s^{(3)}$
3-Particle Basis
⋮
Evolution in n-Particle space determines n-Body component of operator for all A
Original Operator
$\hat{O} = \hat{O}^{(1)} + \hat{O}^{(2)} + \cdots$
$U_s^{(1)}\hat{O}U_s^{(1)}$
1-B Operator
Embed in 2-Particle Basis
$U_s^{(2)}\hat{O}U_s^{(2)}$
$\hat{O}_s^{(1)}$
2-B Operator
Embed in 3-Particle Basis
$U_s^{(3)}\hat{O}U_s^{(3)}$
$\hat{O}_s^{(1)} + \hat{O}_s^{(2)}$
3-B Operator
Embed in A-Particle Basis
A-Particle Calculations

Operator Evolution & Extraction Process

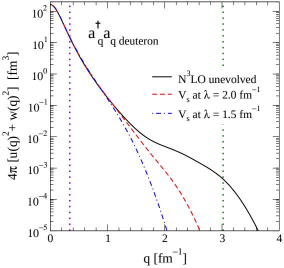


High and Low Momentum operators in the Deuteron

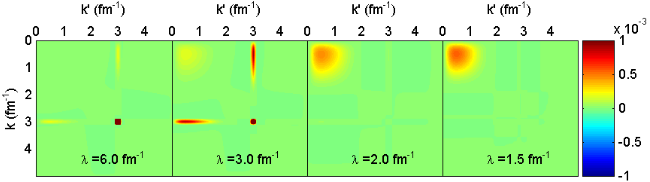
- **Integrand** of $\langle \psi_d | U^\dagger (U a_q^\dagger a_q U^\dagger) U | \psi_d \rangle$ for $q = 0.34\,\mathrm{fm}^{-1}$



- Momentum Distribution



- **Integrand** for $q = 3.02\,\mathrm{fm}^{-1}$



- **Decoupling** $\leftrightarrow$ High momentum components suppressed
- Integrated value does not change, but nature of operator does
- Similar for other operators: $\langle r^2 \rangle$, $\langle Q_d \rangle$, $\langle \frac{1}{r} \rangle$, $\langle G_C \rangle$, $\langle G_Q \rangle$, $\langle G_M \rangle$, etc.

- A=3, 1D model boson system
- **Evolve** Hamiltonian & operators **to** λ **at large** $N_{\max}$

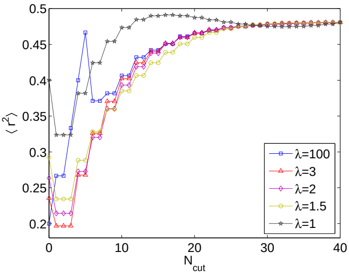
  $\rightarrow$**Truncate** model space at N_{cut}

  $\rightarrow$ Poor convergence of long range operators

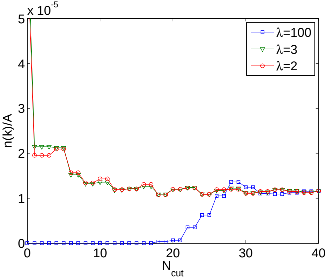
$$\Lambda_{UV} \sim \sqrt{mN_{max}\hbar\omega}; \quad \Lambda_{IR} \sim \sqrt{\frac{m\hbar\omega}{N_{max}}}$$

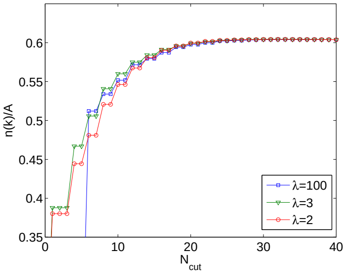
$\longrightarrow$Can this be corrected?

- $\langle r^2 \rangle$ Value



- Number operator at: $q = 15.0$ & $q = 0.5$

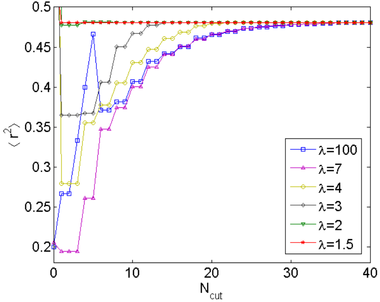




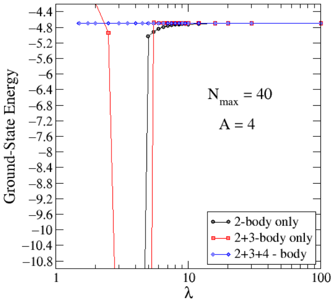
An alternative generator - diagonal in the basis

- The harmonic oscillator Hamiltonian $H_{\rm ho}$ is diagonal in the basis
- Consider a naive choice (replace $T_{\rm rel}$ with $H_{\rm ho}$ in SRG generator)

$$\eta_s = [H_{\rm ho}, H_s] \longrightarrow \frac{dO_s}{ds} = [\eta_s, O_s]$$



$\langle r^2 \rangle$ for A=3



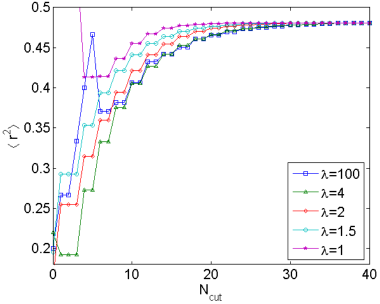
Many Contributions to Binding Energy

- Greatly Improved convergence
- **Generates spurious deep bound states . . .**

Controlled IR and UV renormalization

- Consider $T_{\rm rel} + \alpha r^2$, where α is a parameter which can be adjusted to optimize the renormalization (here, $\alpha = 1$), so that

$$\eta_s = [T_{\rm rel} + \alpha r^2, H_s]$$



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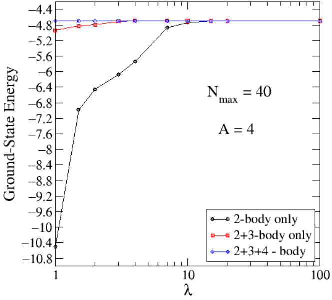
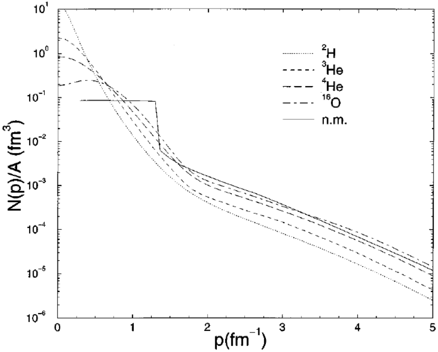


Many Contributions to Binding Energy

- Convergence improves with decreasing λ
- **No spurious deep bound states**. Is hierarchy of many-body forces under control?

Factorization in Few-Body Nuclei

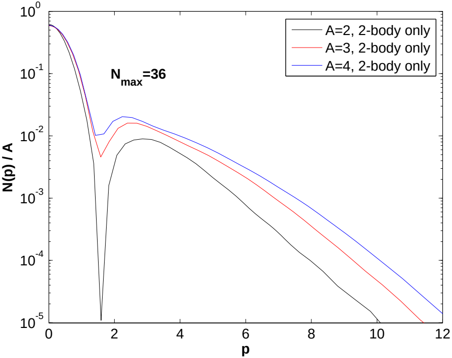
- **Variational Monte Carlo Calculation**
  → Using AV14 NN potential



From Pieper, Wiringa, and Pandharipande (1992).

- Possible explanation of scaling behavior
  → Results from dominance of NN potential and short-range correlations (Frankfurt, et al.)

- **1D few-body HO space calculation**
  → System of A bosons interacting via a model potential



→ A Test Bed for 3D NCSM calculations:

- Alternative explanation of scaling behavior
  → Results from *factorization* . . .

Factorization

$\langle\psi_d| U_\lambda a_q^\dagger a_q U_\lambda^\dagger |\psi_d\rangle$ is independent of λ. What is the nature of $U_\lambda a_q^\dagger a_q U_\lambda^\dagger$?

- From **Decoupling**: write

$$\langle\psi_\lambda| U_\lambda \widehat{O}\, U_\lambda^\dagger |\psi_\lambda\rangle \cong$$

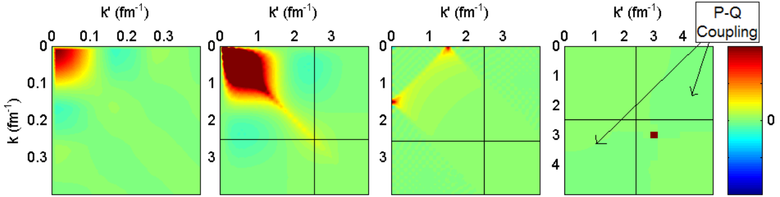
$$\int_0^\lambda dk' \int_0^\infty dq' \int_0^\infty dq \int_0^\lambda dk\ \psi_\lambda^\dagger(k') U_\lambda(k',q') \widehat{O}(q',q) U_\lambda(q,k) \psi_\lambda(k)$$

- Using **Factorization**: set $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$, where $k < \lambda$ and $q \gg \lambda$.

$$\Longrightarrow \int_0^\lambda \int_0^\lambda \psi_\lambda^\dagger(k') \Big[\underbrace{\int_0^\lambda \int_0^\lambda U_\lambda(k',q') \widehat{O}(q',q) U_\lambda(q,k) + I_{QOQ}\, K_\lambda(k') K_\lambda(k)}_{Low\ Momentum\ Structure} \Big] \psi_\lambda(k)$$

where $I_{QOQ} = \int_\lambda^\infty dq' \int_\lambda^\infty dq \left[Q_\lambda(q') \widehat{O}(q',q) Q_\lambda(q) \right]$ $\leftarrow$ *Universal*

– *Valid* when <u>initial</u> operators *weakly couple high and low momentum*, e.g.,



r^2 $\quad \frac{1}{r}$ $\quad G_C(q = 3.02\,\text{fm}^{-1})$ $\quad a_q^\dagger a_q(q = 3.02\,\text{fm}^{-1})$

Factorization in Few-Body Nuclei

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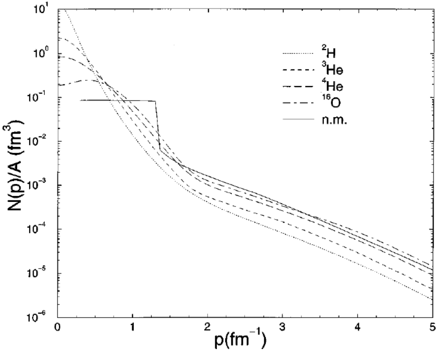
  → Using AV14 NN potential



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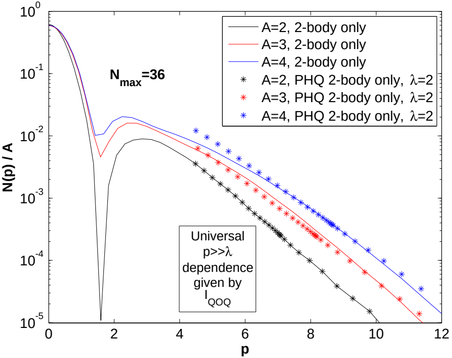

→ A Test Bed for 3D NCSM calculations:

- Alternative explanation of scaling behavior
  → Results from *factorization*

$$\int_0^\lambda \int_0^\lambda \psi_\lambda^\dagger(k') \left[I_{QOQ} K_\lambda(k') K_\lambda(k) \right] \psi_\lambda(k)$$

Conclusion/Outlook

Recent Progress:

- Consistently evolved nuclear operators with SRG in deuteron & model 1D few-body calculations
- Extraction & embedding process for few-body operators formulated and tested
- Explored alternative generators for oscillator basis
- Factorization demonstrated for few-body model calculation

Computational Issues:

- SRG evolution in n-particle basis
  – Exponential growth of matrix size

Plan:

- Establish bounds on growth of many-body operators
- Do calculations in 3D in harmonic oscillator basis
- Explore factorization of other operators (e.g., electroweak)

The End